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Revisit Time-variant Beampatterns for Frequency Diverse Arrays

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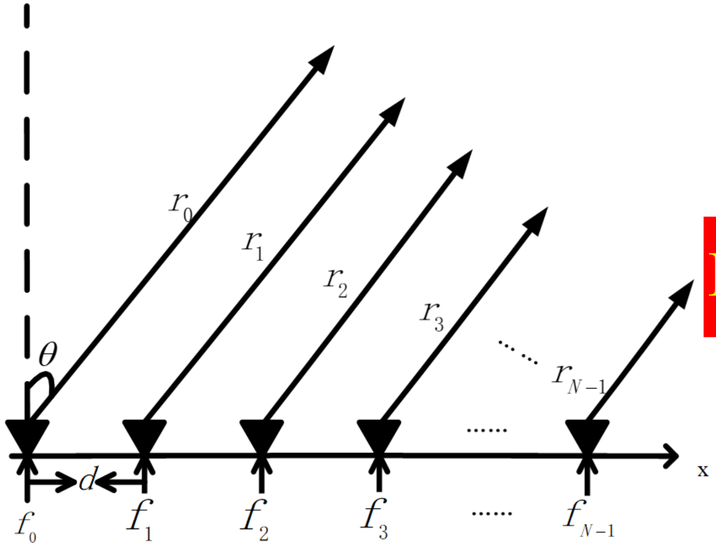
Background (Motivation)

Frequency diverse array (FDA)[1]



$$f_n = f_0 + n\Delta f,\ n = 0, 1 \cdots N-1$$

Fig. 1 Illustration of a 1D FDA

Far-field pattern

$$\mathrm{AF}(t, r\theta) \doteq \sum_{n=0}^{N-1} \exp\left\{-j2\pi f_n\left(t - \frac{r_n}{c}\right)\right\} = \exp\{j\Phi_0\} \sum_{n=0}^{N-1} \exp \left\{-j2\pi\left(n\Delta f t - n\frac{\Delta f r_0}{c} + \frac{m d f_0 \sin\theta}{c} + \frac{m^2 \Delta f d \sin\theta}{c}\right)\right\}$$

$$\approx \exp\{j\Phi_1\} \frac{\sin\left[N\pi\left(\Delta f - \frac{\Delta f r_0}{c} + \frac{d f_0 \sin\theta}{c}\right)\right]}{\sin\left[\pi\left(\Delta f - \frac{\Delta f r_0}{c} + \frac{d f_0 \sin\theta}{c}\right)\right]},$$

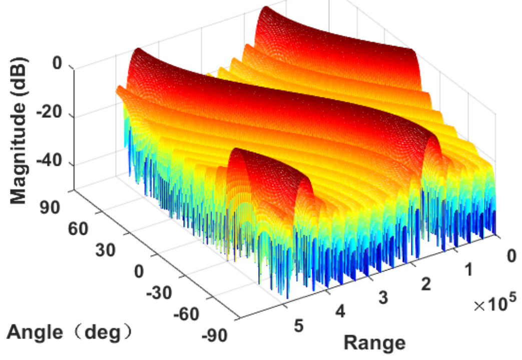
$$r_n \approx r_0 - nd\sin\theta$$

$$d = \frac{c}{2\left(f_0 + (N-1)\cdot|\Delta f|\right)}$$

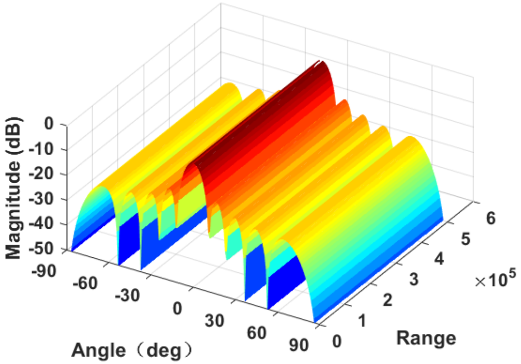
$$\Phi_0 = -2\pi f_0\left(t - (r_0 / c)\right)$$

Characteristics:

- ✓ Automated scanning
- ✓ Range-dependent
- ✓ Range-angle coupling
- ✓ Periodic time modulated pattern in range and angle

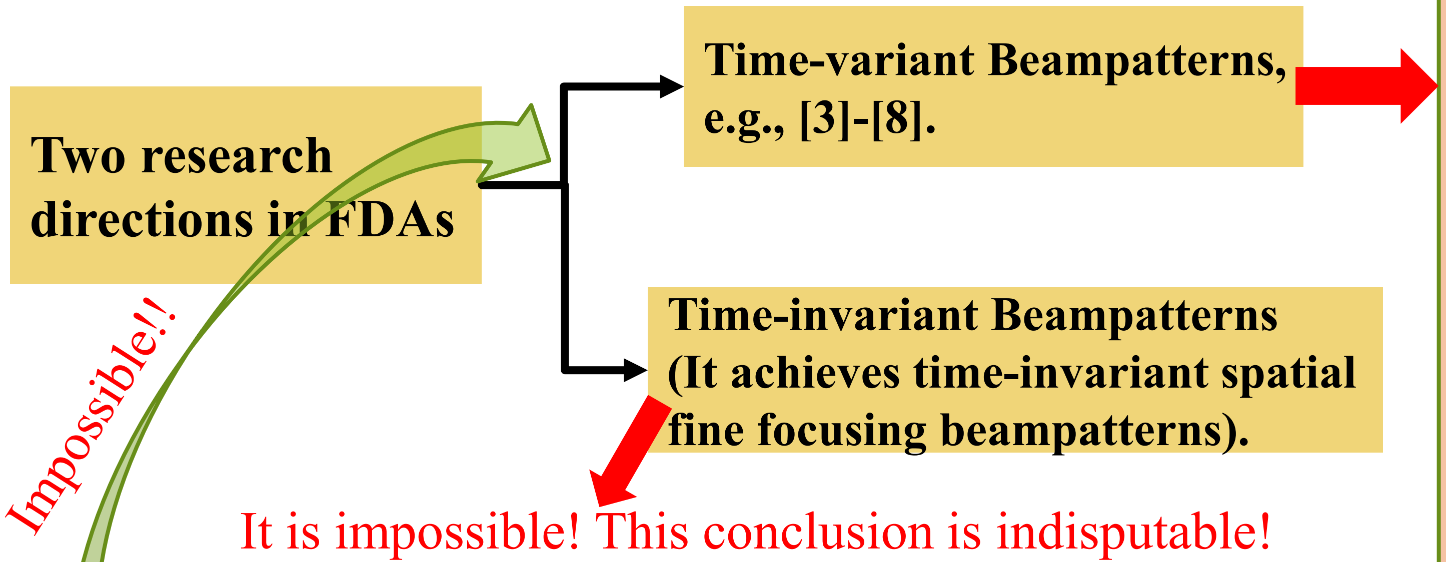


FDA pattern, $\Delta f = 1$ kHz, $N = 10$, $f_0 = 10$ GHz



PA pattern

Background (Motivation)



Two research directions in FDAs

Time-variant Beampatterns, e.g., [3]-[8].

Time-invariant Beampatterns (It achieves time-invariant spatial fine focusing beampatterns).

Impossible!!

It is impossible! This conclusion is indisputable!

In [3] the logarithmically increasing frequency offset FDA, namely log-FDA, was first presented, claiming that an uncoupled range-non-periodic beampattern at the target location can be achieved by way of designing the complex weight associated the radiated signal. This log-FDA is further extended to the multiple-input-multiple-output (MIMO) scenarios in [4], [5], and to the multicarrier waveforms in [6]. In [7], a non-monotonically increasing frequency increment combined with logarithmic offset was described, aiming to obtain improved performance of range-angle localization and reduced sidelobe levels. In [8], the Hamming window-based non-uniform frequency offset for FDA was proposed and compared with the log-FDA, showing that a better signal-to-interference-plus-noise ratio (SINR) performance can be achieved at the desired range-angle location than that does the log-FDA.

[10] B. Chen, X. Chen, Y. Huang, and J. Guan, “Transmit beampattern synthesis for the FDA radar,” *IEEE Antennas Wireless Propag. Lett.* , vol.17, no. 1, pp. 98–101, Jan. 2018.
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Are the time-variant beampatterns of the FDAs claimed in [3]–[8] possible when considering the time-range relationship?

Previously reported time-variant FDAs [3]-[8]

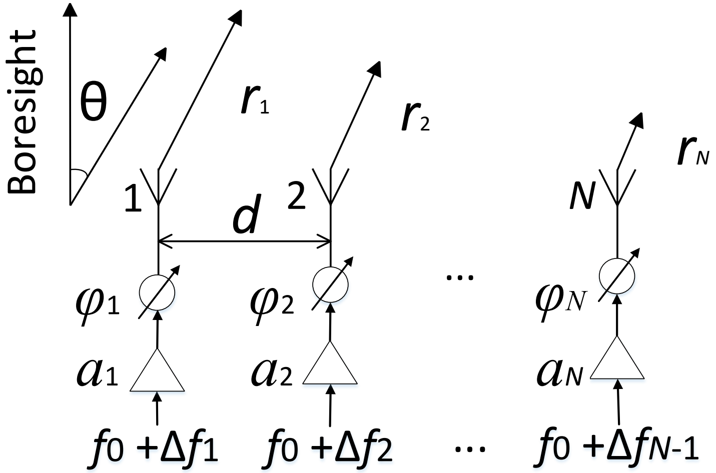


Fig. 2 Illustration of reported time-variant FDA

$$f_n = f_0 + \Delta f_n\,,\quad n = 1, 2, \ldots, N\,. \tag{1}$$

$$S_n(t) = a_n \exp\{j(2\pi f_n t + \varphi_n)\},\quad 0 \le t \le T \tag{2}$$

The transmit beampatterns of the FDAs can be expressed as

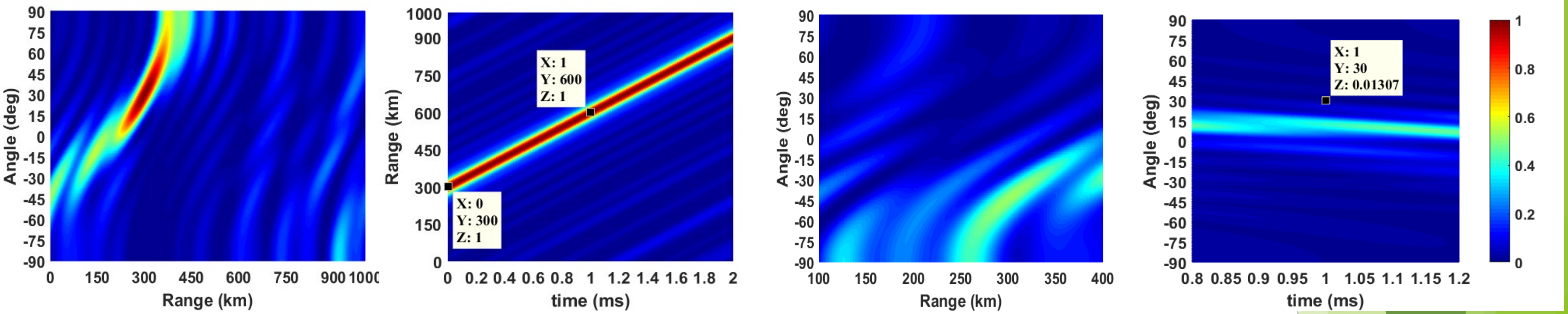
$$\boldsymbol{B}(t; r, \theta) = \left|\sum_{n=1}^{N} \exp\left[j\left(2\pi\Delta f_n\left(t - \frac{r}{c}\right) + \frac{2\pi\left(f_0 + \Delta f_n\right)(n-1)d\sin\theta}{c} + \varphi_n\right)\right]\right|^2 \tag{3}$$

In order to steer the beampattern peak at the target location (r_{x}, θ_{x}), the excitation phase weightings in [3]–[5], [7], [8], were designed as

$$\varphi_n = 2\pi\Delta f_n \frac{r_{\mathrm{x}}}{c} - \frac{2\pi\left(f_0 + \Delta f_n\right)(n-1)d\sin\theta_{\mathrm{x}}}{c} \tag{4}$$

$$\boldsymbol{B}(t; r, \theta) = \left|\sum_{n=1}^{N} \exp\left[j\left(2\pi\Delta f_n\left(t - \frac{r - r_{\mathrm{x}}}{c}\right) + \frac{2\pi\left(f_0 + \Delta f_n\right)(n-1)d\left(\sin\theta - \sin\theta_{\mathrm{x}}\right)}{c}\right)\right]\right|^2 \tag{5}$$

Issues in Previous Time-Variant Beampatterns



(a) (b) (c) (d)

Fig. 3. The normalized transmit beampatterns of the FDA scheme in [3] with (r_x = 300 km, θ_x = 30°) for (a), (c) at time instant t = 0 ms and t = 1 ms, respectively, in the range-angle domains; (b) along angle θ = 30° in the time-range domain; (d) at the distance r_x = 300 km in the time-angle domain.

Parameters: $\Delta f_n = \log(n)\delta$, δ = 2 kHz, f_0 = 5 GHz, d = 0.015 m, N = 10, and T = 2 ms.

The time-range relationship cannot be met !!!

Issues in Previous Time-Variant Beampatterns

What went wrong ???

In (2), the 't' represents the time of the radiated signals and $t = 0$ indicates the signals are about to radiate.

$$S_n(t) = a_n \exp\{j(2\pi f_n t + \varphi_n)\}, \quad 0 \le t \le T \qquad (2)$$

For (5), it corresponds to selecting time reference $t = 0$ when the signals reach the target.

$$\begin{aligned}&\boldsymbol{B}(t; r, \theta)\\&= \left|\sum_{n=1}^{N} \exp\left[j\left(2\pi\Delta f_n \left(t - \frac{r - r_{\mathrm{x}}}{c} \right) + \frac{2\pi (f_0 + \Delta f_n)(n-1) d (\sin\theta - \sin\theta_{\mathrm{x}})}{c} \right)\right]\right|^2\end{aligned} \qquad (5)$$

This mismatch of the selected time references between (2) and (5) leads to essential misconceptions !!!

Rectification

- A. Selecting Time Reference $t = 0$ When the Signals are Radiated from Antenna

In this case, (2) is hold. considering the time-range relationship, the excitation phase weightings should be designed as

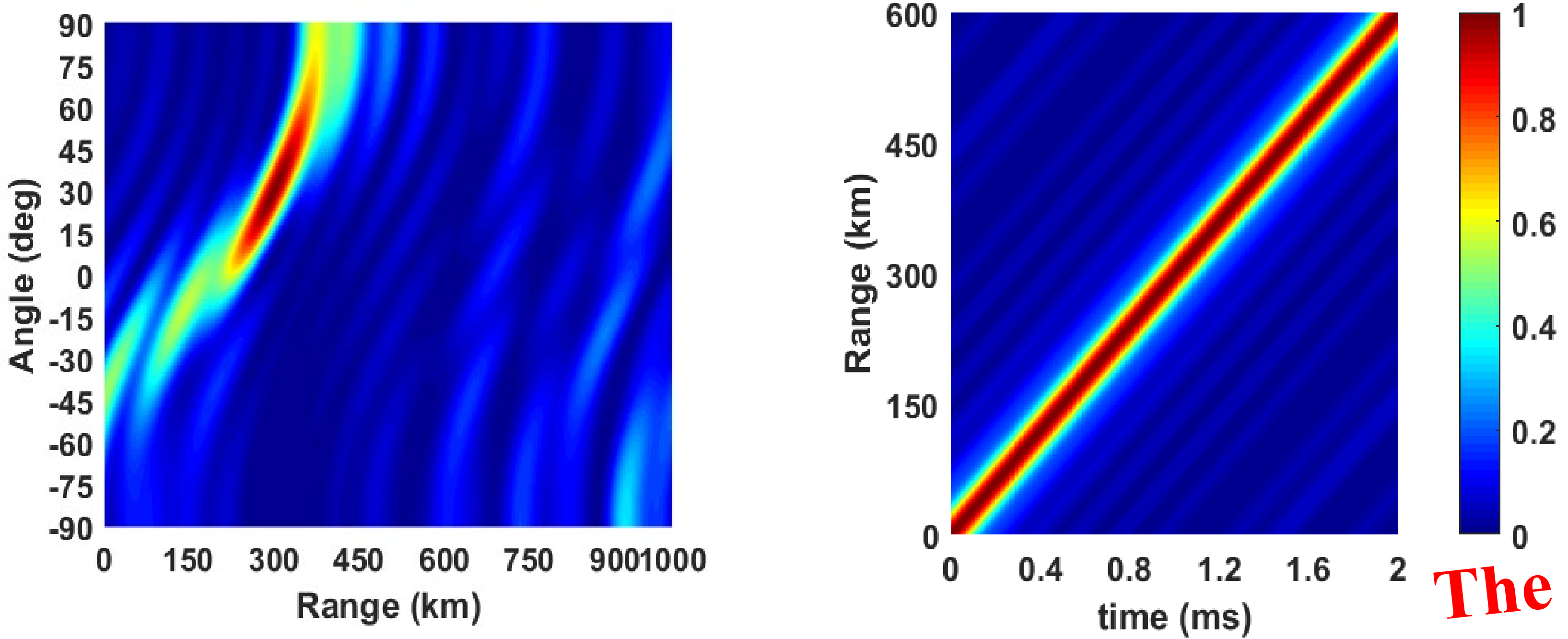
$$\varphi_n = \frac{-2\pi\left(f_0 + \Delta f_n\right)\left(n-1\right) d \sin\theta_{\mathrm{x}}}{c}. \tag{6}$$

Then the transmit beampattern can be expressed as

$$\begin{aligned} &\boldsymbol{B}\left(t; r, \theta\right) \\ &= \left|\sum_{n=1}^{N} \exp\left[j\left(2\pi\Delta f_n \left(t - \frac{r}{c}\right) + \frac{2\pi\left(f_0 + \Delta f_n\right)\left(n-1\right) d\left(\sin\theta - \sin\theta_{\mathrm{x}}\right)}{c} \right)\right]\right|^2. \end{aligned} \tag{7}$$

Rectification

- A. Selecting Time Reference $t = 0$ When the Signals are Radiated from Antenna



(a) (b)

The time-range relationship can be met !

Fig. 4. The normalized transmit beampatterns of the FDA scheme in [3] using the re-designed excitation phase weightings in (6) with $\theta_x = 30°$ for (a) at time instant $t = 1$ ms in the range-angle domain; (b) along angle $\theta = 30°$ in the time-range domain.

Rectification

- B. Selecting Time Reference $t = 0$ When the Signals Reach the Targets

In the case of when the signals reach the targets is selected as the time reference t = 0, the formula (2) needs to be re-defined, which is modified as

$$S_n(t) = a_n \exp\{j(2\pi f_n t + \varphi_n)\} \quad 0 \le \Delta t \le T\,. \tag{8}$$

In (8), ‘t’ denotes the time with the reference ($t = 0$) of when the signals reach the target. Δt represents the time length of the transmitted signals, Under this background, the distance-speed-time formula in free space is equivalent to $r = c\cdot\Delta t$. For instance, at time instant $t = 0$, the distance of the target from the reference antenna $r_x = c\cdot\Delta t = c \cdot |0 - t_s| = -ct_s$, wherein t_s is the time instant when the signals starting to radiate, and it is negative. Hence, when r_x = 300 km, we have $t_s = -1$ ms.

If so, the time-range relationship also can be met !

Conclusions

- The paper proved that the time-range relationship in time-variant FDA beampatterns can be met by way of re-designing the signal excitation phase weightings or re-defining the signal model in previous works.
- The misinterpretation of the time-variant beampatterns rooted in the mismatched time references was revealed.
- **The illustration of the two cases of differently selected time references in this paper are helpful to clear up the confusions in the time variant FDA research community.**

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THANK YOU !

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QUESTIONS?